Comments to the Editor

Bipartite Expressions for Diffusional Mass Transport in Biomembranes

ABSTRACT Analytical expressions for solute diffusion through a membrane barrier for different initial and boundary conditions are available in the literature. The three commonest initial and boundary conditions are for a membrane without solute respectively immersed in a solution of constant concentration, immersed in such a solution for one side but with the other side isolated, and immersed in such a solution for one side and with the other side kept at zero concentration. The physical quantities for the first two initial and boundary conditions are concentration and average concentration (the total solute entering the membrane) with amperometric current (flux) and solute that permeates through the membrane (charge passed) for the third initial and boundary condition. Expressions for these methods in the literature are inconvenient for practical applications because of the infinite mathematical series required. An investigation of convergence of these expressions was therefore carried out. Simple but accurate bipartite expressions for these methods were constructed and provided theoretical support for studies on mass transport characterization of biomembranes. As a specific application, these expressions enabled a direct fit of the simulated observables to experimental values to obtain diffusion coefficients. For these initial and boundary conditions and corresponding physical quantities, simple one point methods for diffusion coefficient estimation are also suggested. These latter diffusion coefficients can be initial values for numerical fit methods.

INTRODUCTION

Solute transport is of fundamental relevance to a broad range of applied scientific areas, including the biomedical and pharmaceutical. As fundamental level diffusional processes are of relevance to transport studies of metabolic, drug, and signaling molecules in tissue culture and across natural tissue boundaries, theoretical models have been developed in parallel with experimental monitoring techniques. Thus, Bashkatov et al. (1) determined diffusion coefficients of glucose and mannitol in human dura mater from time-dependent optical transmittance data, which resulted from local concentration evolution of glucose and mannitol. Gredell et al. (2) determined the diffusion coefficient of propofol through rat brain tissue where propofol concentration was measured by extraction and high-performance liquid chromatography (HPLC). More recently, Goteti et al. (3) determined the diffusion coefficient of dipyridamole in a thermosensitive polymer where dipyridamole concentration was measured using a fluorescence spectrophotometer. Mitchem et al. (4) delivered nitroglycerin through skin mimetics and monitored diffusion by photoacoustic spectroscopy (PAS) and attenuated total reflectance (ATR) spectroscopy. Akimoto et al. (5) determined the diffusion coefficient of indomethacin and antipyrine through rat abdominal skin with HPLC. We measured the diffusion coefficient of electrochemically active acetaminophen and catechol through mixed cellulose esters membranes using planar electrochemical electrodes (6).

The key approach in mass transfer characterization is that a set boundary conditions defines the concentration profiles and therefore changes, which in turn are accessible by experimental techniques, either directly or indirectly (1–6). Concentration evolution in a membrane barrier can be calculated according to Fick's Second Law. By fitting computed concentration profiles or their derivatives to experimental values, numerical quantities, diffusion coefficients can be obtained. Three kinds of initial and boundary conditions are most commonly used for mass transport studies at membranes depending on the analytical solution used and the experimental setup. All, however, are based on concentration C as determined by Fick's Second Law as

$$D\frac{\partial^2 C(x,t)}{\partial x^2} = \frac{\partial C(x,t)}{\partial t}, \tag{1}$$

where D is the diffusion coefficient in cm^2/s; t is time in seconds; and x is the spatial coordinate in centimeters. The solutions to Eq. 1 for three kinds of initial and boundary conditions are analyzed here as follows:

1. A membrane of thickness $2L$ without solute is immersed in solution with concentration C_0; mathematically, $C_1(x,0) = 0$ for $-L < x < L$ and $C_1(-L,t) = C_1(L,t) = C_0$ for $t \geq 0$. The solution to Fick's Second Law is given as (7,8)

$$\frac{C_1(x,t)}{C_0} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\left(\frac{(2n+1)\pi x}{2L}\right) \times \exp\left(-\frac{(2n+1)^2\pi^2 Dt}{4L^2}\right). \tag{2}$$

The integral of Eq. 2 over x gives another physical quantity, average concentration (total solute entering the membrane) as (7,8)

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$$\frac{C_1(t)}{C_0} = 1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left(-\frac{(2n+1)^2\pi^2 Dt}{4L^2}\right). \quad (3)$$

2. A membrane of thickness L without solute, but one side contacts a solution with concentration C_0 and the other side is isolated; mathematically, $C_2(x,0) = 0$ for $0 < x < L$ and $\partial C_2(x,t)/\partial x|_{x=0} = 0$, $C_2(L,t) = C_0$ for $t \geq 0$. The solution to Fick's Second Law is the same as for the first kind of initial and boundary condition, i.e., $C_2(x,t) = C_1(x,t)$, as would be expected from symmetry. Also, the average concentrations are equivalent, i.e., $C_2(t) = C_1(t)$.
3. A membrane of thickness $2L$ without solute, where one side contacts a solution with concentration C_0 and the other side is kept at zero concentration; mathematically, $C_3(x,0) = 0$ for $0 < x < 2L$ and $C_2(0,t) = C_0$, $C_3(2L,t) = 0$ for $t \geq 0$. The solution to Fick's Second Law is given as (7,8)

$$\frac{C_3(x,t)}{C_0} = 1 - \frac{x}{2L} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{2L}\right)\exp\left(-\frac{n^2\pi^2 Dt}{4L^2}\right). \quad (4)$$

The corresponding integral of Eq. 4 over x is half of the integral of Eq. 2 (Eq. 3), i.e., $C_3(t) = C_1(t)/2$.

Under the third initial and boundary condition, two derivative quantities of concentration, i.e., the first derivative of the concentration and an integral of the first derivative, are used for practically characterizing solute mass transport. For a membrane-covered electrochemical sensor (6) responding to a redox active solute, the amperometric current is proportional to solute flux, i.e., the first derivative of concentration to x, $\partial C_3(x,t)/\partial x|_{x=2L}$. The normalized amperometric current (normalized to steady-state current, I_s) is given as (6–8)

$$\frac{I_3}{I_s} = 1 + 2\sum_{n=1}^{\infty}(-1)^n\exp\left(-\frac{n^2\pi^2 Dt}{4L^2}\right). \quad (5)$$

The integral of Eq. 5 over time gives the total charges passed as

$$\frac{I_3 D}{4I_s L^2} = \frac{Dt}{4L^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\exp\left(-\frac{n^2\pi^2 Dt}{4L^2}\right). \quad (6)$$

The total solute permeation through the membrane (5) has the same expression for the dimensionless transient part as the right side of Eq. 6.

A variety of experimental methods have been adopted based on the above theoretical models, i.e., Eqs. 2, 3, 5, and 6 (1–6). Bashkatov et al. (1) calculated the time-dependent optical transmittance of human dura mater due to glucose and mannitol and compared this to experimental values to determine diffusion coefficients. They recognized that Eq. 3 was too complex for practical use and simplified it to

$$C_1(t)/C_0 = 1 - \exp(-\pi^2 Dt/4L^2), \quad (7)$$

where the factor $8/\pi^2$ was omitted for keeping Eq. 7 valid at $t = 0$, i.e., $C_1(0) = 0$. Gredell et al. (2) compared the calculated concentration profiles using a monoexponential function of Eq. 3 to link to the experimental data to determine the diffusion coefficient. They employed a mono- or biexponential function of Eq. 2 to calculate the concentration of propofol as a function of x and t. Goteti et al. (3) fitted Eq. 3 to experimental data to obtain an effective drug diffusion coefficient with special software. Mitchem et al. (4) calculated normalized PAS data for nitroglycerin measurement with an equation similar to Eq. 3 but integral over a smaller distance than membrane thickness, where they summed the first 100 terms of the equation. They also calculated ATR absorbance data using the first 100 terms of Eq. 2 at $x = 0$, subsequently fitting calculated data to experimental values to determine the diffusion coefficient. Akimoto et al. (5) fitted computed values from an equation with the same dimensionless transient part as the right side of Eq. 6 to experimental values to obtain diffusion coefficient values.

In our previous work, an expression for the amperometric currents for small time values were given (6–9) as

$$\frac{I_3}{I_s} = \frac{4L}{(\pi Dt)^{1/2}}\sum_{n=0}^{\infty}\exp\left(-\frac{(2n+1)^2 L^2}{Dt}\right). \quad (8)$$

After an investigation of the convergence of Eqs. 5 and 8, a simple bipartite expression for amperometric current was constructed as (6,9)

$$I_3/I_s = 1 - 2\exp(-\pi^2 Dt/4L^2) \quad \text{for} \quad t \geq 0.2369(2L)^2/D, I_3/I_s \geq 0.8072 \quad (9a)$$

$$I_3/I_s = 4L/(\pi Dt)^{1/2}\exp(-L^2/Dt) \quad \text{for} \quad t \leq 0.2369(2L)^2/D, I_3/I_s \leq 0.8072. \quad (9b)$$

Equation 9 was used to directly fit calculated currents to experimental values to more accurately determine the diffusion coefficient (6). A preliminary value for the diffusion coefficient was estimated from $T = Dt_1/L^2 = 0.5$, i.e., when the transient current reaches 0.9856 of the steady-state current. In this study, we select a more suitable point to estimate the diffusion coefficient, which is also suitable for other methods.

Previous improper simplified expressions or expressions with infinite series have held up reliable experimental data processing, so we analyze the convergence of Eqs. 2, 3, 4, 6, and their complementary counterparts for small time values

and then construct the corresponding bipartite expressions. These simple but accurate expressions are proposed here as a generalizable tool for reliable mass transport characterization in membranes with specific capability for determining the diffusion coefficient by fitting simulated data to experimental data.

MATHEMATICAL ANALYSIS

Solute concentration under the first kind of initial and boundary conditions, i.e., Eq. 2, is a function of two variables, x and t. Initially Eq. 2 at $x = 0$ is analyzed, and for a simplified mathematical analysis, dimensionless time $T = Dt/4L^2$ is introduced. Eq. 2 at $x = 0$ is written as

$$\frac{C_1(0,T)}{C_0} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\exp[-(2n+1)^2\pi^2 T]. \quad (10)$$

The terms of Eq. 10 corresponding to $n = 0$, 1, and 2 are shown in Fig. 1 *a*. It can be seen from Fig. 1 *a* that the absolute value of the term decreases with increasing n or T and this tendency is also true for $n > 2$. For $T > 0.06$, the terms with $n > 0$ become negligible. Eq. 10 with n limited to 0, 1, and 2 are represented in Fig. 1 *b* for comparison. It can be seen that Eq. 10 with n limited to 0 can represent the whole solution to Fick's Second Law because the curves with different terms are overlapping for $T > 0.06$. In other words, Eq. 10 converges rapidly for large T, but slowly for small T, and it is therefore inappropriate to use Eq. 10 for numerical calculation for small T. Fortunately, a solution of Eq. 1 for small T is available as (7,8)

$$\frac{C_1(x,t)}{C_0} = \sum_{n=0}^{\infty}(-1)^n \times \left[\mathrm{erfc}\left(\frac{(2n+1)L-x}{2(Dt)^{1/2}}\right) + \mathrm{erfc}\left(\frac{(2n+1)L+x}{2(Dt)^{1/2}}\right)\right], \quad (11)$$

where erfc is an error function (10). At $x = 0$, Eq. 11 is written as

$$\frac{C_1(0,T)}{C_0} = 2\sum_{n=0}^{\infty}(-1)^n\mathrm{erfc}\left(\frac{2n+1}{4T^{1/2}}\right). \quad (12)$$

The terms of Eq. 12 corresponding to $n = 0$, 1, and 2 are shown in Fig. 2 *a*. It can be seen from Fig. 2 *a* that the absolute value of the term of Eq. 12 increases with T increasing and decreases with n increasing; this tendency is also true for $n > 2$. Eq. 12 with n limited to 0, 1, and 2 is shown in Fig. 2 *b* for comparison. For $T < 0.10$, the terms with $n > 0$ are negligible; therefore, the first term of Eq. 12 represents the whole solution as the curves with different terms overlap.

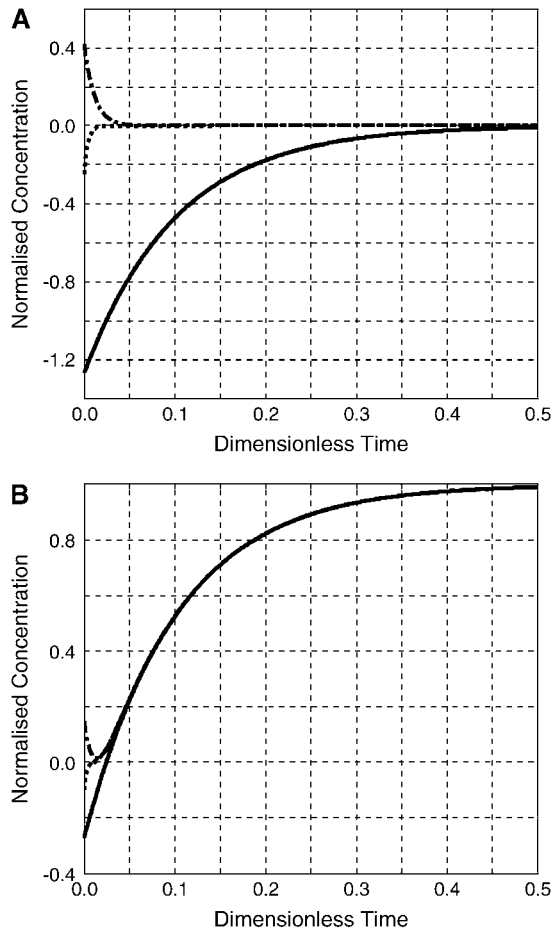

FIGURE 1 (*a*) Normalized concentration versus dimensionless time for terms of Eq. 10 in the text, corresponding to $n = 0$ (*solid line*), 1 (*dash-dot line*), and 2 (*dotted line*); (*b*) normalized concentration versus dimensionless time for Eq. 10 in the text, with n limited to 0 (*solid line*), 1 (*dash-dot line*), and 2 (*dotted line*).

RESULTS AND DISCUSSION

From the above analysis, it is possible to construct a solution function that combines the rapid convergent parts of Eq. 10 and Eq. 12. For a smooth connection, the two function curves are required to intersect and to have the same value or close values for the first derivatives at the intersection point. After balancing accuracy versus complexity, a function was constructed as follows:

$$C_1(0,T)/C_0 = 1-(4/\pi)\exp(-\pi^2 T) \quad \text{for} \quad T \geq 0.07958, \quad C_1(0,T)/C_0 \geq 0.4199 \quad (13a)$$

$$C_1(0,T)/C_0 = 2\mathrm{erfc}(1/4T^{1/2}) \quad \text{for} \quad T \leq 0.07958, \quad C_1(0,T)/C_0 \leq 0.4199. \quad (13b)$$

The rapid convergent parts of Eq. 10 and Eq. 12 overlap, and therefore a complete solution function, Eq. 13, can be

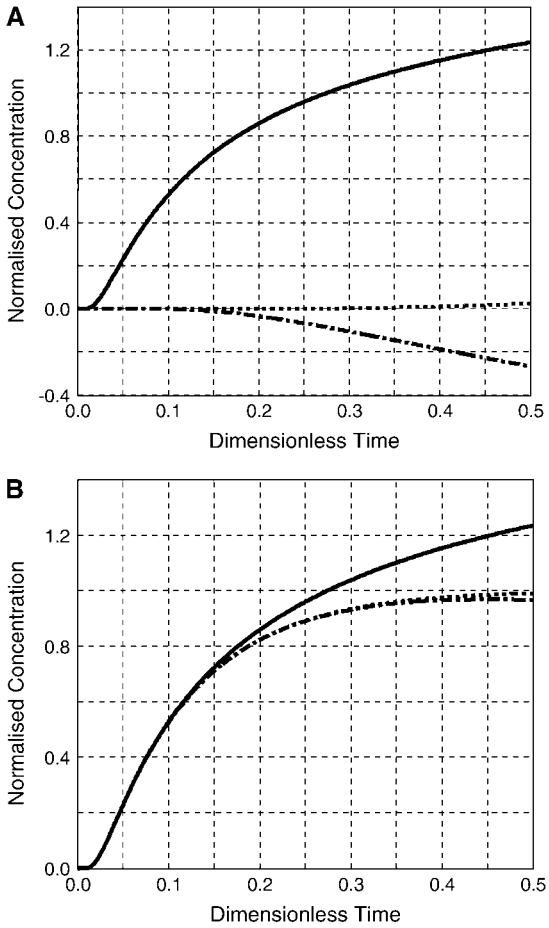

FIGURE 2 (*a*) Normalized concentration versus dimensionless time for terms of Eq. 12 in the text, corresponding to $n = 0$ (*solid line*), 1 (*dash-dot line*), and 2 (*dotted line*); (*b*) normalized concentration versus dimensionless time for Eq. 12 in the text, with n limited to 0 (*solid line*), 1 (*dash-dotted line*), and 2 (*dotted line*).

constructed, with the expressions cut into fast and slow convergent parts, respectively, and the two fast convergent parts then joined to form a practical function for an entire time range. At the joint point ($T = 0.07958$), the absolute values of terms of Eq. 10 corresponding to $n = 0$, 1, 2, 3, and 4 are 0.5805, 3.613×10^{-4}, 7.556×10^{-10}, 3.513×10^{-18}, and 3.320×10^{-29}, respectively, which decreases dramatically with n increasing. This tendency is also true for $n > 4$. Therefore, the terms with $n > 0$ are negligible for $T > 0.07958$. At the joint point, the absolute values of the first five terms of Eq. 12 are 0.4202, 3.400×10^{-4}, 7.386×10^{-10}, 3.478×10^{-18}, and 3.308×10^{-29}, respectively, which decreases dramatically with n increasing. This tendency is again also true for $n > 4$. The terms with $n > 0$ are negligible for $T < 0.07958$. Therefore, it is reasonable that only the term with n limited to 0 of Eq. 10 and the first term of Eq. 12 are sufficient to construct the full solution function, i.e., Eq. 13. From the values of Eq. 10 and Eq. 12 at the joint point, the maximum error of the proper simplified expression, i.e., Eq. 13 is $<0.04\%$.

The diffusion coefficient can be estimated from time $t_{0.5}$ when concentration (Eq. 10 or Eq. 12) reaches half of steady-state value as

$$D = 0.09469(2L)^2/t_{0.5}. \tag{14}$$

After the bipartite expression for concentration at $x = 0$, i.e., $C(0,t)$ was constructed, a bipartite expression for the concentration function, $C(x,t)$, can be constructed with the same accuracy and joint point as Eq. 13. The bipartite expression consists of Eq. 2 with n limited to 0 for $T \geq 0.07958$ and Eq. 11 with n limited to 1 for $T \leq 0.07958$. Because $\cos(x) \leq 1$, n is limited to 0 for Eq. 2 whereas n is limited to 1 for Eq. 11 because of a variable domain in the error function. Explicitly, the terms with the absolute values larger than $\mathrm{erfc}(3/4T^{1/2})$ are retained.

For average concentration defined by Eq. 3, a solution for small t is given as (7,8)

$$\frac{C_1(t)}{C_0} = \frac{2}{L}\left(\frac{Dt}{\pi}\right)^{1/2} + \frac{4(Dt)^{1/2}}{L}\sum_{n=1}^{\infty}(-1)^n \mathrm{ierfc}\left(\frac{nL}{(Dt)^{1/2}}\right), \tag{15}$$

where $\mathrm{ierfc}(y)$ is the integral of the error function $\mathrm{erfc}(y)$ and $\mathrm{ierfc}(y) = \exp(-y^2)/\pi^{1/2} - y\mathrm{erfc}(y)$ (10). Similarly a bipartite expression can be constructed as follows (11):

$$C_1(T)/C_0 = 1-(8/\pi^2)\exp(-\pi^2 T) \quad \text{for} \quad T \geq 0.05326, \quad C_1(T)/C_0 \geq 0.5200 \tag{16a}$$

$$C_1(T)/C_0 = 4(T/\pi)^{1/2} \quad \text{for} \quad T \leq 0.05326, \quad C_1(T)/C_0 \leq 0.5200. \tag{16b}$$

At the joint point ($T = 0.05326$), the absolute values of the terms of Eq. 3 corresponding to $n = 0$, 1, 2, 3, and 4 are 0.4792, 7.942×10^{-4}, 6.362×10^{-8}, 1.078×10^{-13}, and 3.228×10^{-21}, respectively. At the joint point, the absolute values of the terms of Eq. 15 corresponding to $n = 1$, 2, 3, 4, and 5 are 7.946×10^{-4}, 1.807×10^{-10}, 5.359×10^{-21}, 1.643×10^{-35}, and 4.762×10^{-54}, respectively, which thus decreases dramatically with n increasing. From the values of Eq. 3 and Eq. 15 at the joint point, the maximum error of the final simplified expression, i.e., Eq. 16, is $<0.08\%$.

For a one point method, the diffusion coefficient can, in principle, be relatively easily estimated from time $t_{0.5}$ when average concentration (Eq. 3 or Eq. 15) reaches half of steady-state concentration (11) as

$$D = 0.04918(2L)^2/t_{0.5}. \tag{17}$$

In the case of concentration under the third kind of initial and boundary condition, i.e., Eq. 4, the complementary counterpart expression for small time values is given as

$$\frac{C_3(x,t)}{C_0} = \sum_{n=0}^{\infty}\left[\operatorname{erfc}\left(\frac{4nL+x}{2(Dt)^{1/2}}\right) - \operatorname{erfc}\left(\frac{4(n+1)L-x}{2(Dt)^{1/2}}\right)\right]. \tag{18}$$

By comparison of Eq. 2 with Eq. 4, and Eq. 11 with Eq. 18, respectively, it is found that $C_3(L,t) = C_1(0,t)/2$. A bipartite expression for $C_3(L,t)$ can be constructed as for $C_1(0,t)$ with the same joint point, terms used, and accuracy. Subsequently, a bipartite expression for $C_3(x,t)$ can be constructed by combining Eq. 4 with n limited to 2 for $T \geq 0.07958$ and Eq. 18 with n limited to 0 for $T \leq 0.07958$. It should be pointed out that Eq. 4 with $n = 2$ is needed and Eq. 18 with only $n = 0$ is sufficient in the construction because of variable domains in the functions.

In our previous work, a bipartite expression for amperometric current was constructed (6,9); there, the corresponding one point method can be improved. The diffusion coefficient can be determined from time $t_{0.5}$ when transient current (Eq. 5 or Eq. 8) reaches half of steady-state current as

$$D = 0.1388(2L)^2/t_{0.5}. \tag{19}$$

Equation 19 provides a more direct and precise method to estimate the diffusion coefficient because the transient current curve is steeper at $I_3/I_s = 0.5$ than at $I_3/I_s = 0.9856$; therefore, the time when the current reaches half of steady-state current is likely to be more accurately estimated.

For the charge passed in the case of an electrochemical sensor used to monitor the flux of an electroactive compound (Eq. 6), the complementary counterpart expression of charge passed for small time values is given as (7)

$$\frac{I_3 D}{4I_s L^2} = \frac{2(Dt)^{1/2}}{L}\sum_{n=0}^{\infty}\operatorname{ierfc}\left(\frac{(2n+1)L}{(Dt)^{1/2}}\right). \tag{20}$$

Similarly a bipartite expression can be constructed from Eq. 6 and Eq. 20 as

$$I_3 D/(4I_s L^2) = T - 1/6 + (2/\pi^2)\exp(-\pi^2 T) \quad \text{for} \quad T \geq 0.2369,$$
$$I_3 D/(4I_s L^2) \geq 0.08979 \tag{21a}$$

$$I_3 D/(4I_s L^2) = 4T^{1/2}\operatorname{ierfc}(1/2T^{1/2}) \quad \text{for} \quad T \leq 0.2369,$$
$$I_3 D/(4I_s L^2) \leq 0.08979. \tag{21b}$$

At the joint point ($T = 0.2369$), the absolute values of the terms of Eq. 6 corresponding to $n = 1$, 2, 3, 4, and 5 are 0.01956, 4.395×10^{-6}, 1.635×10^{-11}, 7.174×10^{-19}, and 3.335×10^{-28}, respectively. At the joint point, the first five terms of Eq. 20 are 0.08978, 3.788×10^{-6}, 6.678×10^{-14}, 3.604×10^{-25}, and 4.757×10^{-40}, respectively. From the values of Eqs. 6 and 20 at the joint point, the maximum error of the final simplified expression, i.e., Eq. 21, is 0.0004%.

Because of the lack of a steady-state value for the charge passed (total solute permeated), there is another way to estimate the diffusion coefficient. If Eq. 6 is plotted, i.e., total charge versus time, the asymptotic line intersects the time axis at t_L, and the diffusion coefficient can then be estimated as (12)

$$D = (2L)^2/(6t_L). \tag{22}$$

This is the well-known time lag method and was first defined by Dynes (12). The diffusion coefficient estimated from Eq. 22 can be refined by fitting Eq. 21 to the experimental data.

Above, various bipartite expressions are constructed and one point methods developed. By way of illustration a bipartite expression is used to analyze previous work. Bashkatov et al. (1) used a simplified expression, Eq. 7, for averaged concentration, but this led to a source error, the difference between Eq. 7 and Eq. 16 versus dimensionless time, as shown in Fig. 3. According to the analysis of dura mater tissue samples used by Bashkatov et al. (1), $D = 1.31 \times 10^{-6}$ cm^2/s, $2L = 0.43$ mm, 0.52 mm, 0.65 mm for mannitol, and $D = 1.63 \times 10^{-6}$ cm^2/s, $2L = 0.52$ mm, 0.56 mm, 0.59 mm for glucose, the corresponding dimensionless times for $t = 15$ min measurement time are $T = 0.64$, 0.44, and 0.28 for mannitol and $T = 0.54$, 0.47, and 0.42 for glucose. The error distribution in Fig. 3

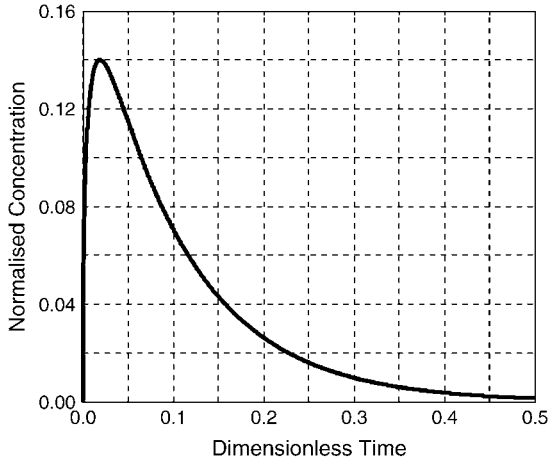

FIGURE 3 The error of Eq. 7 in the text versus dimensionless time, i.e., difference between Eq. 7 and Eq. 16 in the text.

shows that the maximum error is up to 14%. The mean errors of Eq. 7 were obtained by averaging the curve in Fig. 3 over 15 min (the corresponding dimensionless time is different in each case because of a different diffusion coefficient and tissue sample thickness), and the average error values are 2.8%, 4.0%, and 6.0% for mannitol diffusion in 0.43 mm, 0.52 mm, and 0.65 mm dura mater, and 3.3%, 3.8%, and 4.2% for glucose diffusion in 0.52 mm, 0.56 mm, and 0.59 mm dura mater. Membrane swelling was not taken into account at the times the average errors of Eq. 7 were calculated. However, swelling of a tissue sample will increase tissue thickness and therefore will reduce the dimensionless time further to increase the average error (Fig. 3) even further. The error in average concentration would have led to an error in calculated optical transmittance, which was therefore likely to be greater than the 1% suggested. These workers actually fitted calculated (transmittance) curves to a data range of experimental curves within one standard deviation, whereas conventionally for an experiment with such errors, the defined experimental value is considered to be precisely at the statistical mean value, not any free standard deviation data range around a mean. There is, thus, less reliability in the curve fitting method, particularly when, as reported, the mean transmittance value changes during two experiments were less than the standard deviation range for glucose and mannitol measurements. A more appropriate fitting approach would be to best fit the calculated optical transmittance with a bipartite expression (Eq. 16) for average concentration to the experimental mean values. Curve fitting here essentially involves a baseline shift or a curvature change to the calculated curve by iterative adjustment of the diffusion coefficient. The curves reported by Bashkatov et al. (1) can in fact be shifted and their curvature changed to improve the fit, providing more accurate diffusion coefficients with a best match to the observed data.

If instead of Eq. 7, a monoexponential, Eq. 3 with $n = 0$ approximates Eq. 3, the maximum error (difference between Eq. 3 with $n = 0$ and Eq. 13) occurs at $t = 0$ as 18.9%. Thus, only the bipartite expression can provide a simple but accurate solution.

Mitchem et al. (4) used the first 100 terms of Eq. 10 to calculate the ATR absorbance data. The 101st term at $t = 0$ equals 0.006 and gives the order of the maximum error. Therefore, the accuracy of Eq. 10 summing the first 100 terms is still one order of magnitude lower than that of the bipartite equation as Eq. 13, so again a bipartite expression is preferred.

CONCLUSION

For studies on diffusional mass transport through membrane barriers with initially no solute in the membrane, three kinds of boundary conditions are commonly used, in which one side of the membrane contacts a solution with constant solute concentration, C_0, whereas the other contacts the same solution, is isolated or is kept at zero concentration, respectively. Accurate, but simple, bipartite expressions have been constructed for concentration and average concentration (total solute entering the membrane) for three boundary conditions and amperometric current (flux) and charge passed (solute permeated through the membrane) for the third boundary condition. The nature of bipartite expression is that the slow convergent part of an expression is replaced by a fast convergent part of its complementary counterpart expression. So the expression is simple whereas accuracy is retained. The error of a bipartite expression can be estimated from the values of the original expression at its joint point. The term of the bipartite expression used was based upon a practical assumption of an accuracy of 0.1%. The accuracy of a bipartite expression can, in fact, be improved upon by increasing the number of terms. For an ideal system a bipartite expression and one point method are equivalent for diffusion coefficient determination. Because of experimental errors and theoretical approximation, a one point method, though simple, may contain greater errors. However, a one point method is suggested for an initial estimate of the diffusion coefficient, which can be refined by direct fit of the calculated observables to experimental data. The direct fit method with the bipartite expressions has general application to mass transport studies in biophysical systems, where a one-dimensional model is valid and shown to improve reliability in a measurement field notoriously susceptible to high experimental variability.

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